

REPORT NO. NAWCADWAR-95012-4.3

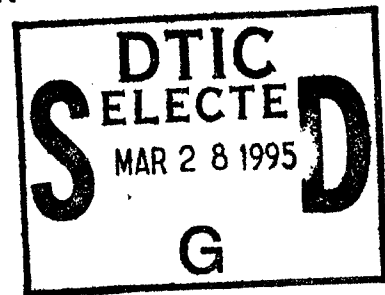


## EVALUATION OF PROPOSED AGILITY METRICS USING X-31 VS. F/A-18 FLIGHT DATA

Julieta E. Booz  
Air Vehicle Department (Code 432300R08)  
NAVAL AIR WARFARE CENTER AIRCRAFT DIVISION  
Warminster, PA 18974-0591

30 NOVEMBER 1994

FINAL REPORT  
Period Covering October 1993 to September 1994



*Approved for Public Release; Distribution is Unlimited.*

Prepared for  
OFFICE OF NAVAL RESEARCH  
Ballston Center Tower 1  
800 North Quincy Tower 1  
Arlington, VA 22217-5000

19950327 069

DTIC QUALITY INSPECTED 1

## NOTICES

**REPORT NUMBERING SYSTEM** - The numbering of technical project reports issued by the Naval Air Warfare Center, Aircraft Division, Warminster is arranged for specific identification purposes. Each number consists of the Center acronym, the calendar year in which the number was assigned, the sequence number of the report within the specific calendar year, and the official 2-digit correspondence code of the Functional Department responsible for the report. For example: Report NO. NAWCADWAR-92001-4.3 indicates the first Center report for the year 1992 and prepared by the Air Vehicle Department. The numerical codes are as follows:

CODE	FUNCTIONAL DEPARTMENT
4.1	Systems Engineering Department
4.2	Cost Analysis Department
4.3	Air Vehicle Department
4.4	Propulsion and Power Department
4.5	Avionics Department
4.6	Crew Systems Engineering Department
4.7	Weapons and Subscale Targets Department
4.8	Support Equipment and ALRE Department
4.9	Training Systems Department
4.10	Conc Analysis, Evaluation, and Planning (CAEP) Department
4.11	Test and Evaluation Engineering Department
4.12	Ship/Shore Electronic Systems Department

**PRODUCT ENDORSEMENT** - The discussion or instructions concerning commercial products herein do not constitute an endorsement by the Government nor do they convey or imply the license or right to use such products.

Reviewed By: Robert E Palmer Date: 1/30/95  
Branch Head

Reviewed By: John A Clark Jr Date: 1/31/95  
Division Head

Reviewed By: \_\_\_\_\_ Date: \_\_\_\_\_  
Department Head

# REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE NOVEMBER 1994	3. REPORT TYPE AND DATES COVERED FINAL - OCTOBER 1993-SEPTEMBER 1994	
4. TITLE AND SUBTITLE EVALUATION OF PROPOSED AGILITY METRICS USING X-31 VS. F/A-18 FLIGHT DATA			5. FUNDING NUMBERS	
6. AUTHOR(S)  JULIETA E. BOOZ				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  AIR VEHICLE DEPARTMENT (CODE 432300R08)			8. PERFORMING ORGANIZATION REPORT NUMBER  NAWCADWAR-95012-4.3	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) OFFICE OF NAVAL RESEARCH BALLSTON CENTER TOWER 1 800 NORTH QUINCY STREET ARLINGTON VA 22217-5000			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  This report is an evaluation of the specific agility metrics and tactical agility metrics as described in Veda Technical Report 33158-94U/P3553-003, "Aircraft Fighter Agility: Theory, Advanced Metrics, and Simulation." The agility metrics were applied to six engagements flown between the X-31 and the F/A-18. A comparison was done to determine if the agility metrics correlated with the results from the engagements flown (e.g. did the more agile aircraft get the "win"). The results indicate that applying these agility metrics to correlate with the win or loss of an engagement did not provide consistent predictions as to which aircraft would score a win. Evaluation of the data from the engagements indicate that what determined which aircraft achieved the win (by obtaining a missile kill) was that aircraft which was the first to maneuver into an offensive position within the weapons firing solution. This was primarily done by generating the higher acceleration rate ( $\dot{n}_z$ ) or by obtaining a higher initial acceleration ( $n_z$ ) value.				
14. SUBJECT TERMS  AGILITY,X-31,F/A-18,PERFORMANCE,AGILITY METRICS			15. NUMBER OF PAGES	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT	

## SUMMARY

This report is an evaluation of the specific agility metrics and tactical agility metrics as described in Veda Technical Report 33158-94U/P3553-003, "Aircraft Fighter Agility: Theory, Advanced Metrics, and Simulation." The agility metrics were applied to six engagements flown between the X-31 and the F/A-18. A comparison was done to determine if the agility metrics correlated with the results from the engagements flown (e.g. did the more agile aircraft get the "win"). The results indicate that applying these agility metrics to correlate with the win or loss of an engagement did not provide consistent predictions as to which aircraft would score a win. Evaluation of the data from the engagements indicate that what determined which aircraft achieved the win (by obtaining a missile kill) was that aircraft which was the first to maneuver into an offensive position within the weapons firing solution. This was primarily done by generating the higher acceleration rate ( $\dot{n}_z$ ) or the obtaining a higher initial acceleration ( $n_z$ ) value.

Accession For	
NTIS	CRA&I <input checked="" type="checkbox"/>
DTIC	TAB <input type="checkbox"/>
Unannounced <input type="checkbox"/>	
Justification _____	
By _____	
Distribution / _____	
Availability Codes	
Dist	Avail and/or Special
A-1	

## TABLE OF CONTENTS

	Page
SUMMARY	i
LIST OF FIGURES	iii
LIST OF SYMBOLS	iv
1.0 INTRODUCTION	1
1.1 PURPOSE	1
1.2 BACKGROUND	1
1.2.1 AGILITY METRICS	1
1.2.2 CLOSE-IN-COMBAT TACTICAL ENGAGEMENTS	2
2.0 APPROACH	7
3.0 RESULTS	7
3.1 COMPARISON WITH AGILITY METRICS	7
3.2 OBSERVATIONS	15
4.0 CONCLUSIONS	21
5.0 RECOMMENDATIONS	21
REFERENCES	22
APPENDIX A - FRENET'S FORMULAS AS APPLIED TO THE ACCELERATION RATE VECTORS	A-1
APPENDIX B - TABULATED DATA	B-1

## LIST OF FIGURES

	Page
Figure 1. Defensive Starting Condition for X-31 Tactical Engagement	3
Figure 2. Slow Speed Line-Abreast Starting Condition for X-31 Tactical Engagement	3
Figure 3. High Speed Line-Abreast Starting Condition for X-31 Tactical Engagement	3
Figure 4. 3-D Plot of Engagement 139SSLA	4
Figure 5. 3-D Plot of Engagement 142SSLA	4
Figure 6. 3-D Plot of Engagement 180HSLA	5
Figure 7. 3-D Plot of Engagement 195HSLA	5
Figure 8. 3-D Plot of Engagement 198DEF	6
Figure 9. 3-D Plot of Engagement 197DEF	6
Figure 10. Specific Axial Agility Metric - Full Engagement Time	8
Figure 11. Specific Curvature Agility Metric - Full Engagement Time	8
Figure 12. Specific Torsional Agility Metric - Full Engagement Time	8
Figure 13. Specific Total Agility Metric - Full Engagement Time	9
Figure 14. Tactical Axial Agility Metric - Full Engagement Time	9
Figure 15. Tactical Curvature Agility Metric - Full Engagement Time	10
Figure 16. Tactical Torsional Agility Metric - Full Engagement Time	10
Figure 17. Tactical Total Agility Metric - Full Engagement Time	10
Figure 18. Velocity for Engagement 198DEF	11
Figure 19. Specific Axial Agility Metric - Partial Engagement Time	12
Figure 20. Specific Curvature Agility Metric - Partial Engagement Time	12
Figure 21. Specific Torsional Agility Metric - Partial Engagement Time	12
Figure 22. Specific Total Agility Metric - Partial Engagement Time	13
Figure 23. Tactical Axial Agility Metric - Partial Engagement Time	13
Figure 24. Tactical Curvature Agility Metric - Partial Engagement Time	14
Figure 25. Tactical Torsional Agility Metric - Partial Engagement Time	14
Figure 26. Tactical Total Agility Metric - Partial Engagement Time	14
Figure 27. Off-Boresight Angle Plot of Engagement 139SSLA	16
Figure 28. Off-Boresight Angle Plot of Engagement 142SSLA	16
Figure 29. Off-Boresight Angle Plot of Engagement 180HSLA	16
Figure 30. Off-Boresight Angle Plot of Engagement 195HSLA	17
Figure 31. Off-Boresight Angle Plot of Engagement 198DEF	17
Figure 32. Off-Boresight Angle Plot of Engagement 197DEF	17
Figure 33. $N_z$ & $\dot{N}_z$ Plot of Engagement 139SSLA	18
Figure 34. $N_z$ & $\dot{N}_z$ Plot of Engagement 142SSLA	18
Figure 35. $N_z$ & $\dot{N}_z$ Plot of Engagement 180HSLA	19
Figure 36. $N_z$ & $\dot{N}_z$ Plot of Engagement 195HSLA	19
Figure 37. $N_z$ & $\dot{N}_z$ Plot of Engagement 198DEF	19
Figure 38. $N_z$ & $\dot{N}_z$ Plot of Engagement 197DEF	20
Figure 39. $N_z$ Plot of Engagement 197DEF for Full Engagement	20

## LIST OF SYMBOLS

$\Delta t$	delta time (sec)
$\eta$	number of inflection points
$\kappa$	flight path curvature
$\tau$	flight path torsion
$\Lambda_b$	Tactical Torsional Agility ( $\text{ft}^2/\text{sec}^4$ )
$\Lambda_n$	Tactical Curvature Agility ( $\text{ft}^2/\text{sec}^4$ )
$\Lambda_t$	Tactical Axial Agility ( $\text{ft}^2/\text{sec}^4$ )
$\Lambda_{\text{Total}}$	Total Tactical Agility ( $\text{ft}^2/\text{sec}^4$ )
$\dot{a}_b$	Torsional Agility Rate Vector Component ( $\text{ft}/\text{sec}^3$ )
$\dot{a}_n$	Curvature Agility Rate Vector Component ( $\text{ft}/\text{sec}^3$ )
$\dot{a}_t$	Axial Agility Rate Vector Component ( $\text{ft}/\text{sec}^3$ )
CIC	close-in-combat
DEF	defensive setup
HSLA	high speed line abreast setup
KCAS	knots calibrated airspeed (kts)
$n_z$	body-axis normal acceleration (g)
$\dot{n}_x$	body-axis longitudinal acceleration rate (g/sec)
$\dot{n}_y$	body-axis lateral acceleration rate (g/sec)
$\dot{n}_z$	body-axis normal acceleration rate (g/sec)
$P_s$	specific excess power (ft/sec)
$\dot{P}_{\text{stab}}$	stability axis roll acceleration ( $\text{deg}/\text{sec}^2$ )
$p$	roll rate ( $\text{deg}/\text{sec}$ )
$q$	pitch rate ( $\text{deg}/\text{sec}$ )
$r$	yaw rate ( $\text{deg}/\text{sec}$ )
rms	root mean square
$s$	arc length of flight path (ft)
$\dot{s}$	first derivative of the flight path wrt time ( $\text{ft}/\text{sec}$ )
$\ddot{s}$	second derivative of the flight path wrt time ( $\text{ft}/\text{sec}^2$ )
$\dddot{s}$	third derivative of the flight path wrt time ( $\text{ft}/\text{sec}^3$ )
$S$	total flight path length (ft)
SSLA	slow speed line abreast setup

## 1.0 INTRODUCTION

This effort is in support of the exploratory block program entitled "Air Vehicle Technology Block", Section 4.0: "Dynamics of Flight", Task Area 4.6: "Flight Dynamics - Fighter Agility". This work documents the effort to correlate previously developed agility metrics with data from tactical engagements between the X-31 experimental aircraft and an F/A-18 fighter/attack aircraft. Also addressed in this report are observations of trends noted between the different engagements.

### 1.1 PURPOSE

The purpose of this effort was to evaluate how applicable the agility metrics proposed in Veda Technical Report "Aircraft Fighter Agility: Theory, Advanced Metrics, and Simulation" (reference 1) are when applied to the combat scenario. To date much of the effort to develop agility metrics has been theoretical and applied to a single aircraft executing a single maneuver. This effort is an attempt to better understand the applicability of the proposed metrics and potentially identify any limitations of the proposed metrics.

### 1.2 BACKGROUND

#### 1.2.1 AGILITY METRICS

The agility metrics proposed in reference 1 are called, Specific and Tactical Agility. They are a function of the Acceleration Rate Vectors (ARV) components which are derived from the Frenet's formulas (reference 2). Frenet's formulas describe a curve in space (or for this application the flight path of an aircraft) as a function of curvature ( $\kappa$ ) and torsion ( $\tau$ ). In order to define curvature and torsion one needs to take the third derivative with respect to time, hence the development of the ARV components. A more thorough discussion of Frenet's Formulas and the ARV components is provided in Appendix A.

Reference 1 defines the Specific Agility metrics as:

$$\text{Specific Axial Agility} = (\dot{a}_t)_{\text{rms}} = (\ddot{s} - \dot{s}^3 \kappa^2)_{\text{rms}} \quad (\text{ft/sec}^3) \quad (1)$$

$$\text{Specific Curvature Agility} = (\dot{a}_n)_{\text{rms}} = (3\ddot{s}\dot{s} + \dot{s}^2 \dot{\kappa})_{\text{rms}} \quad (\text{ft/sec}^3) \quad (2)$$

$$\text{Specific Torsional Agility} = (\dot{a}_b)_{\text{rms}} = (\dot{s}^3 \kappa \tau)_{\text{rms}} \quad (\text{ft/sec}^3) \quad (3)$$

$$\text{Specific Total Agility} = \left( \sqrt{\dot{a}_t^2 + \dot{a}_n^2 + \dot{a}_b^2} \right)_{\text{rms}} \quad (\text{ft/sec}^3) \quad (4)$$



where:

rms = the root mean square of each ARV component over the duration of the selected flight path.

The Tactical Agility Metrics are defined in reference 1 as:

$$\text{Tactical Axial Agility} = \Lambda_t = \eta S(\dot{a}_t)_{\text{rms}} / \Delta t \text{ (ft}^2/\text{sec}^4) \quad (5)$$

$$\text{Tactical Curvature Agility} = \Lambda_n = \eta S(\dot{a}_n)_{\text{rms}} / \Delta t \text{ (ft}^2/\text{sec}^4) \quad (6)$$

$$\text{Tactical Torsional Agility} = \Lambda_b = \eta S(\dot{a}_b)_{\text{rms}} / \Delta t \text{ (ft}^2/\text{sec}^4) \quad (7)$$

$$\text{Tactical Total Agility} = \Lambda_{\text{Total}} = \eta S(\dot{a})_{\text{rms}} / \Delta t \text{ (ft}^2/\text{sec}^4) \quad (8)$$

where:

$\eta$  = the number of inflection points appearing in the selected flight path segment and/or +1 for every 180 degree turn in either circular or helical-like segments.

$S$  = the total arc length of the flight path (ft)

$\Delta t$  = the duration of the selected flight path (sec)

## 1.2.2 CLOSE-IN-COMBAT TACTICAL ENGAGEMENTS

For this effort three (3) close-in-combat (CIC) starting conditions for the tactical engagements were used (reference 3). They are shown in figures 1 through 3. The defensive (DEF) setup has the X-31 in a defensive position from the F/A-18 approximately 3000 feet apart with both aircraft flying at approximately 325 KCAS. This was usually achieved by having one aircraft turning so that it can be perpendicular to the other aircraft at approximately the desired setup speed and distance. The slow speed line abreast (SSLA) has the two aircraft in a neutral position 1500 feet abreast at 215 KCAS. The third scenario evaluated was the high speed line abreast (HSLA) again the two aircraft are in a neutral position but at 3000 feet abreast at 325 KCAS.

Data were made available for 18 different CIC engagements (reference 4). From those engagements two cases with the X-31 and the F/A-18 each scoring a win from each scenario were evaluated. This was done to try to determine if one aircraft was more agile than the other for that particular engagement. It was also done to try to determine what, if any, one thing could predict whether an aircraft would win or lose an engagement.

Figures 4 though 9 show the 3-D plots of the 6 engagements investigated in this study.

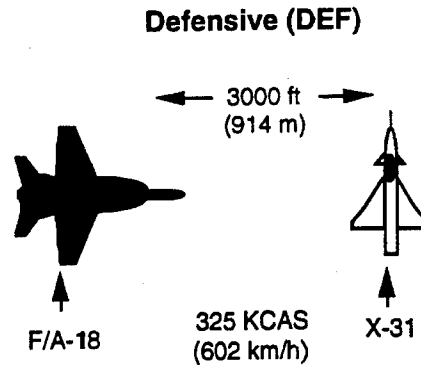


Figure 1. Defensive Starting Condition for X-31 Tactical Engagement

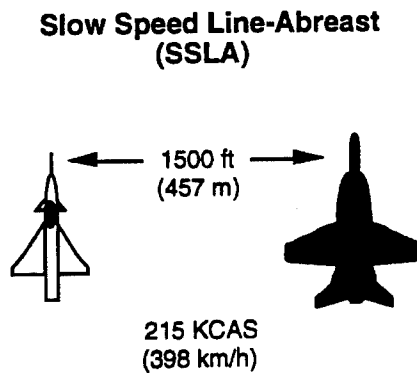


Figure 2. Slow Speed Line-Abreast Starting Condition for X-31 Tactical Engagement

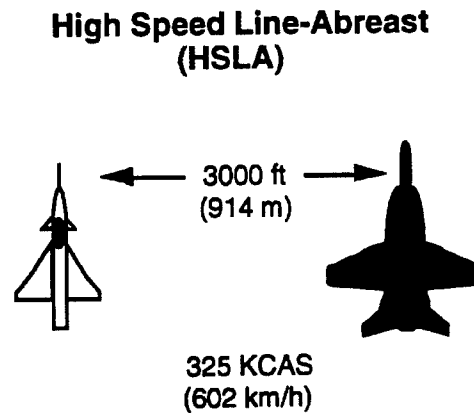


Figure 3. High Speed Line-Abreast Starting Condition for X-31 Tactical Engagement

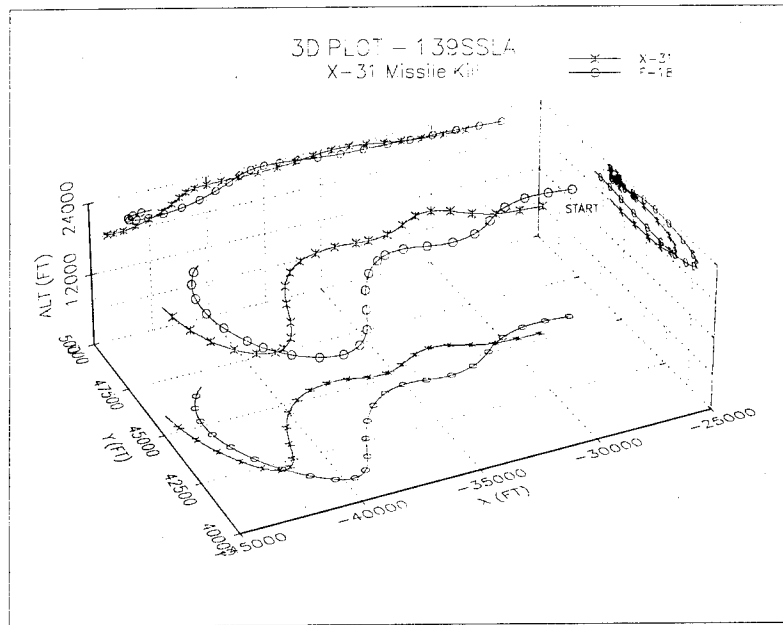


Figure 4. 3-D Plot of Engagement 139SSLA

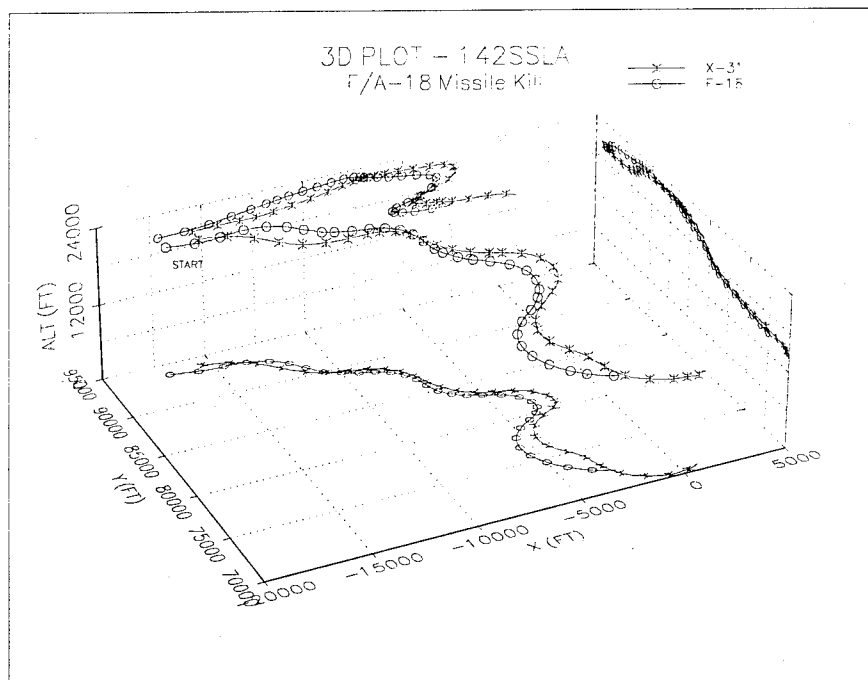


Figure 5. 3-D Plot of Engagement 142SSLA

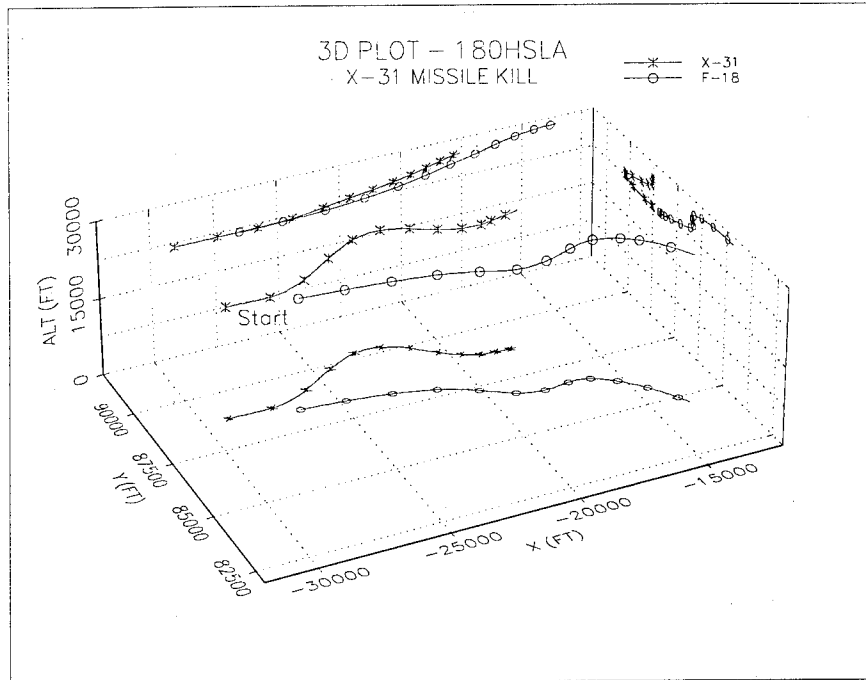


Figure 6. 3-D Plot of Engagement 180HSLA

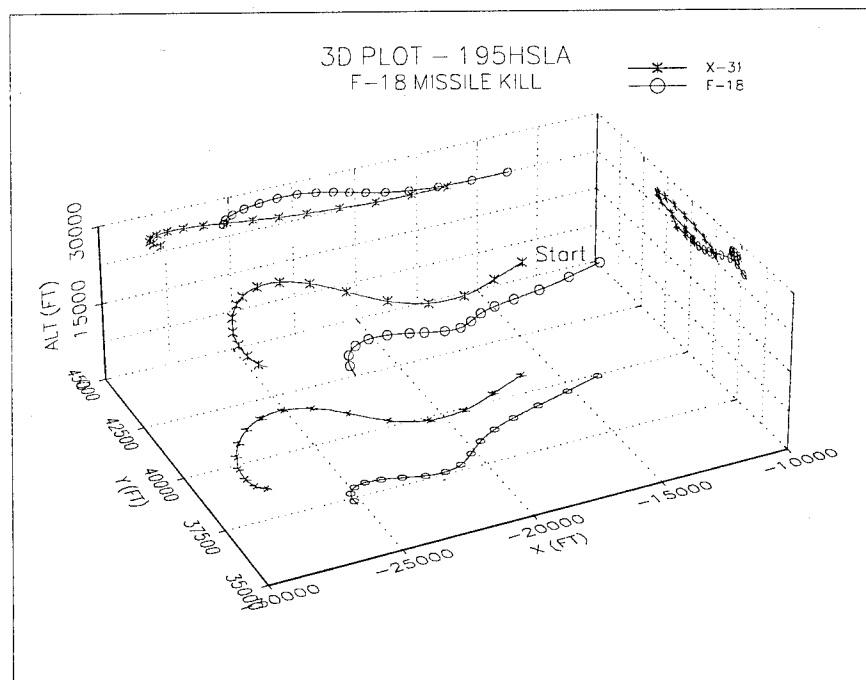


Figure 7. 3-D Plot of Engagement 195HSLA

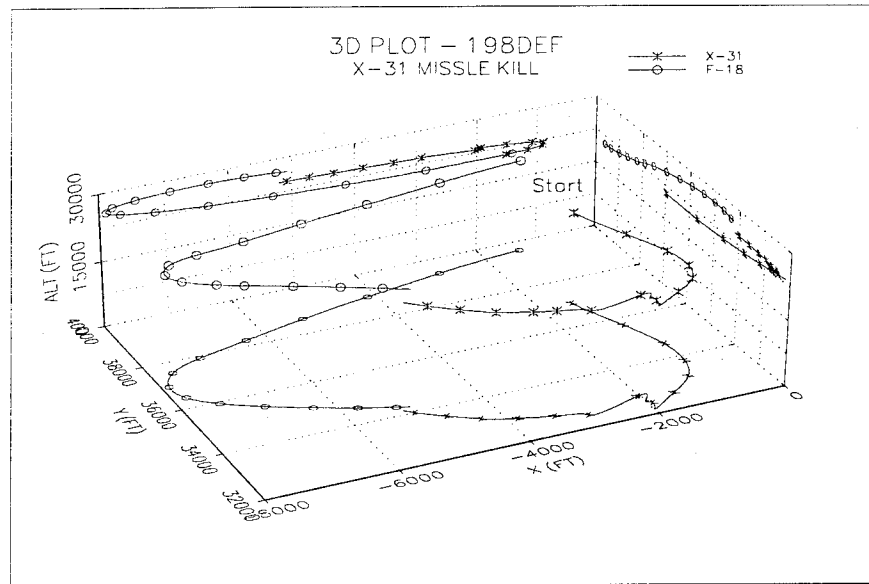


Figure 8. 3-D Plot of Engagement 198DEF

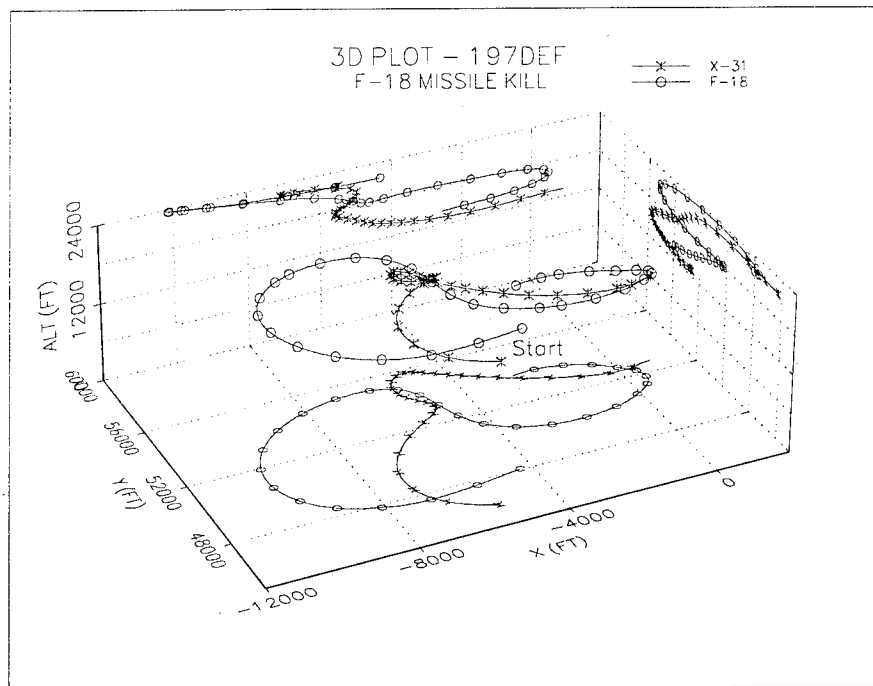


Figure 9. 3-D Plot of Engagement 197DEF

## 2.0 APPROACH

Reference 1 theorizes that the aircraft with the larger value for Specific and Tactical Agility would be more agile and hence "win" an engagement. The approach taken in this effort was to apply the Specific and Tactical Agility Metrics to the six (6) tactical engagements between the X-31 and F/A-18. Given the same initial Tactical scenario the X-31 and the F-18 each scored a "win" by killing the opposing aircraft with a missile shot. The resulting values for the agility metrics were then compared with the engagement outcomes to determine if this premise was valid. In addition to applying the agility metrics over the whole course of the engagements, the time span was reduced to the point where the winning aircraft was in position to fire the missile for the kill, approximately 30 seconds into the engagement.

Although the Tactical Agility Metrics included the  $\eta$  term, much of the data presented in reference 1 did not include  $\eta$  in the presentation of its results. Since one aircraft was essentially tracking the other aircraft, or they were doing a scissors-like maneuver with the same number of reversals, the  $\eta$  was also not included in this analysis to maintain consistency in computing the Tactical Agility Metrics.

In addition to evaluating the proposed agility metrics, aircraft performance parameters such as velocity,  $n_x$ ,  $\dot{n}_z$ , pitch and roll rate were also reviewed to determine if they could provide additional insight as to which aircraft was more agile.

## 3.0 RESULTS

The following sections describe the results of applying the agility metrics and general observations found for the six engagements used in this study. Tabulated data for the results are presented in Appendix B. Performance plots for the six engagements are presented in Appendix C. Also recall that, as mentioned in Section 2.0, the  $\eta$  term was not included in the calculation of the Tactical Agility metrics.

### 3.1 COMPARISON WITH AGILITY METRICS

Figures 10 through 13 show the Specific Agility metrics for the total time frame of each of the six CIC scenarios. As can be seen in all four figures, having the higher magnitude for each of the metrics did not necessary guarantee that the more agile aircraft as defined by these metrics would get the win. In figures 11 through 13 the X-31 did have the higher values for each of the metrics (Specific Curvature, Specific Torsional and Specific Total Agility) but only won half of the engagements. In figure 10 the X-31's Specific Axial Agility for the SSLA scenario was less than that of the F/A-18's and won in one of the SSLA cases. In engagement 197DEF the difference between the X-31 and the F/A-18 Specific Axial Agility was only  $0.004 \text{ ft/sec}^3$  with the F/A-18 having the slightly lesser value and still scoring a win.

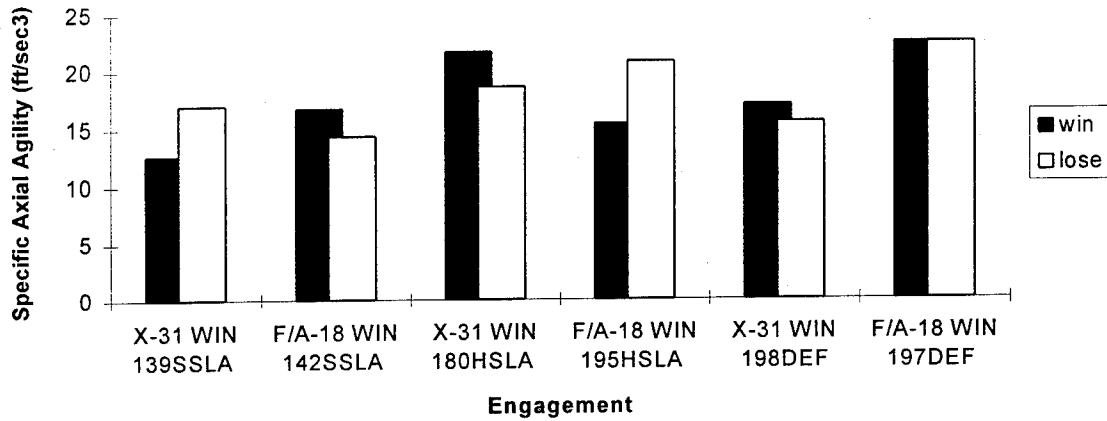


Figure 10. Specific Axial Agility Metric - Full Engagement Time

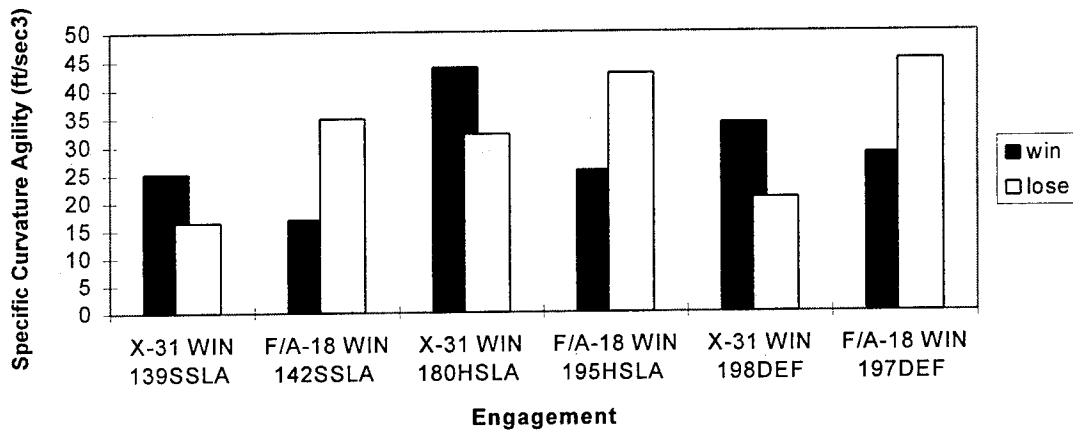


Figure 11. Specific Curvature Agility Metric - Full Engagement Time

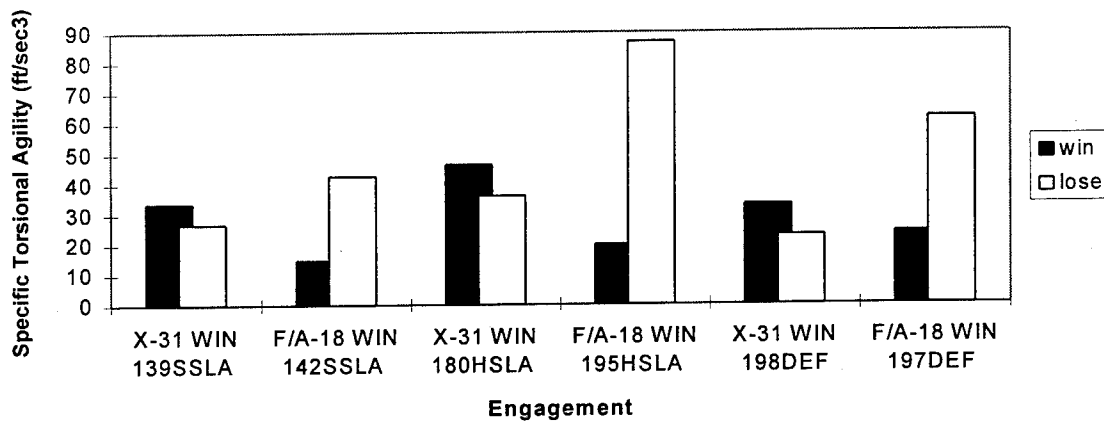


Figure 12. Specific Torsional Agility Metric - Full Engagement Time

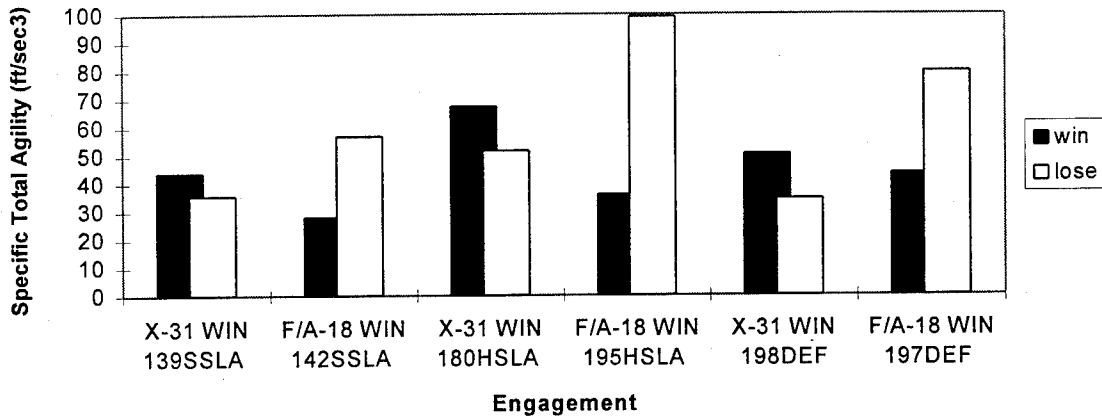


Figure 13. Specific Total Agility Metric - Full Engagement Time

Figures 14 through 17 show the Tactical Agility metrics for the total time frame of each of the six CIC engagements. As with the Specific Agility metrics computations, the results were mixed with half the winning engagements showing higher values for the Tactical Agility metrics. In only one engagement (197DEF) did the F/A-18 consistently show higher values for the Tactical Agility metrics. For that same set-up where the X-31 achieved the win (198DEF) the X-31 demonstrated a lower Tactical Agility metric value than the F/A-18. Although for that engagement (198DEF) the X-31 Specific Agility metric values were always greater than the F/A-18 values. A possible explanation is that the Tactical Agility metrics incorporates an average speed term ( $S/\Delta t$ ). When the X-31 takes advantage of its post-stall capabilities, its speed is less than the F/A-18 as shown in figure 18. Hence the lower value of the Tactical Agility metrics.

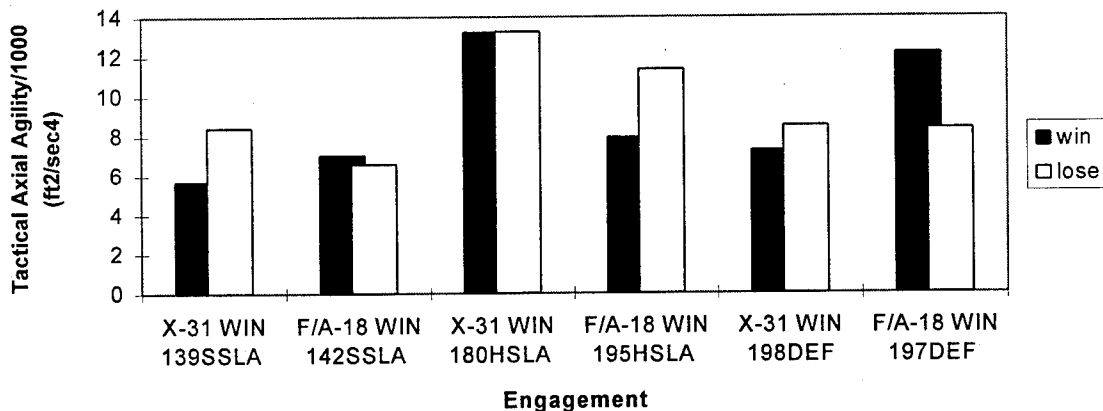


Figure 14. Tactical Axial Agility Metric - Full Engagement Time



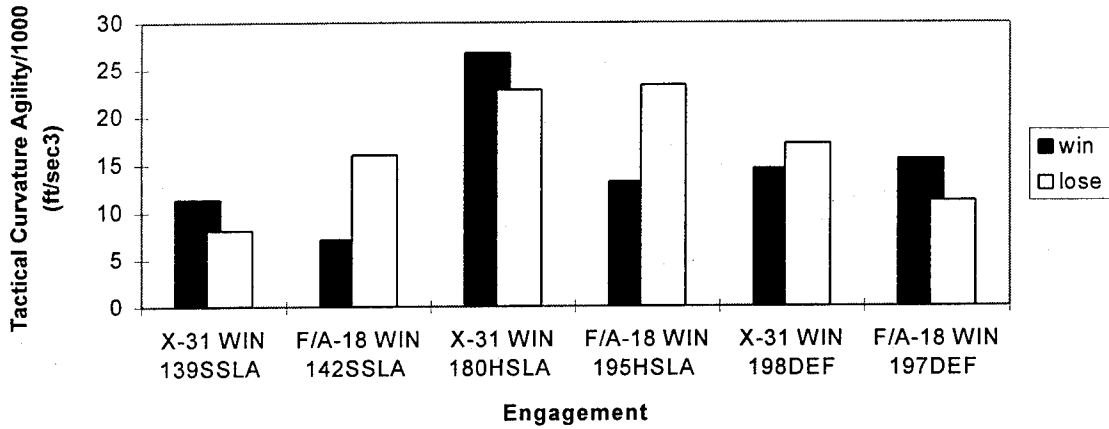


Figure 15. Tactical Curvature Agility Metric - Full Engagement Time

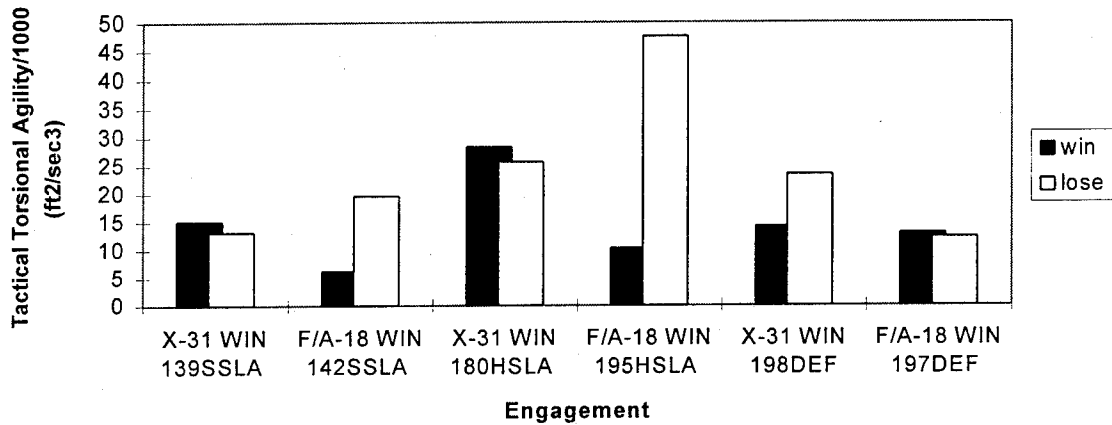


Figure 16. Tactical Torsional Agility Metric - Full Engagement Time

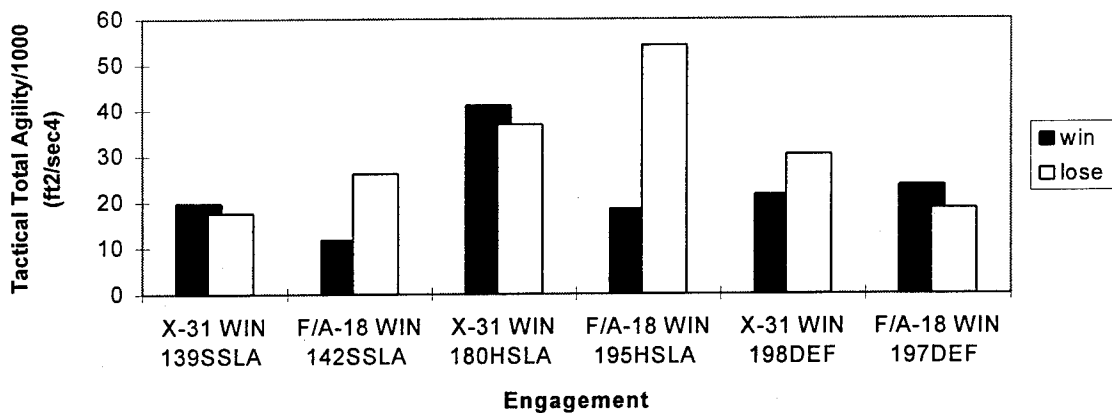


Figure 17. Tactical Total Agility Metric - Full Engagement Time

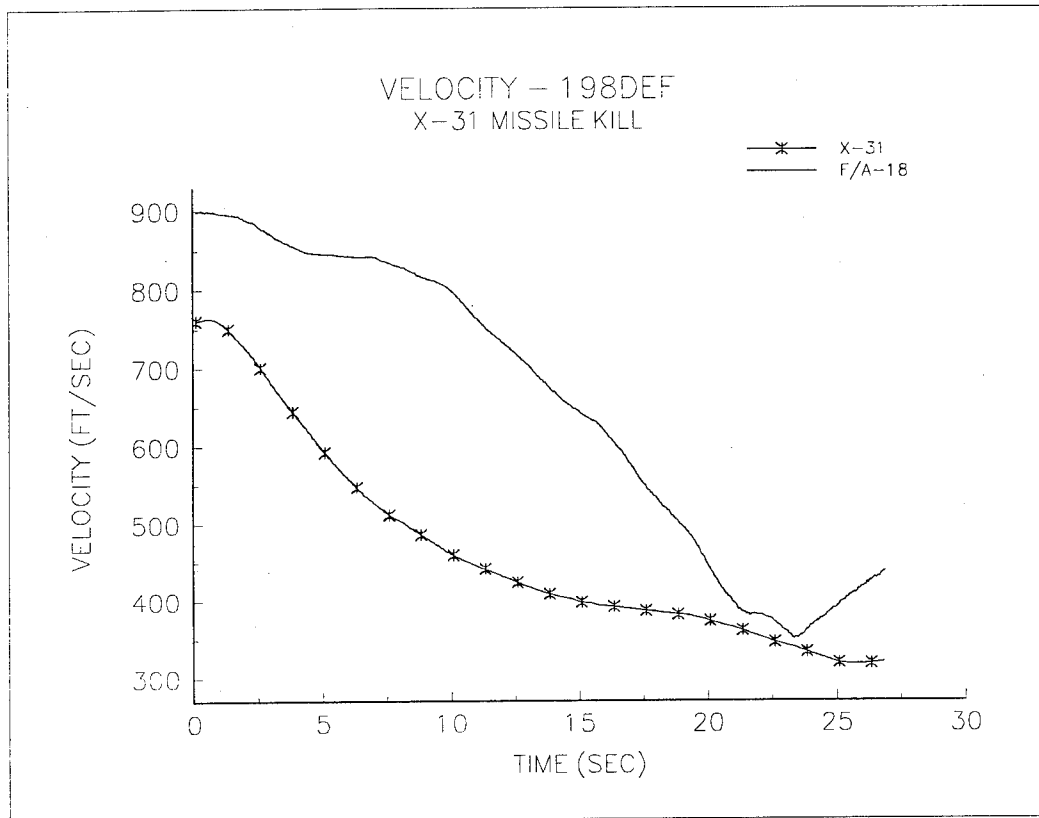


Figure 18. Velocity for Engagement 198DEF

Upon reviewing the various performance plots generated for the CIC engagements it was found that in many instances the defensive aircraft had higher turn rates, pitch and roll rates and generally executed more abrupt maneuvers than the offensive aircraft. It was thought that this behavior was possibly contributing to the disparity in the agility metrics. Reviewing the data plots showed that approximately half way through the engagement the aggressor aircraft generally had the advantage by being within a weapons firing solution and maintained that advantage throughout the maneuver. The approach taken was then to reduce the engagement time of interest for the six engagements. For the HSLA and SSLA scenarios a time frame of 30 seconds was used. The 198DEF total engagement time was less than 30 seconds so the time frame of interest for that engagement was reduced to 15 seconds.

Figures 19 through 22 are the resulting Specific Agility metric values for the partial time frames. The results did not demonstrate an appreciable improvement as compared with using the full engagement time for computing the Specific Agility Metrics. In figure 19 an additional X-31 (139SSLA) win was indicated as compared to the Specific Axial Agility results in figure 10. There was no difference in the results for Specific Curvature Agility as shown in figure 20. In figures 21 and 22 the predictive capability of the agility metrics were worse than using the full engagement time.

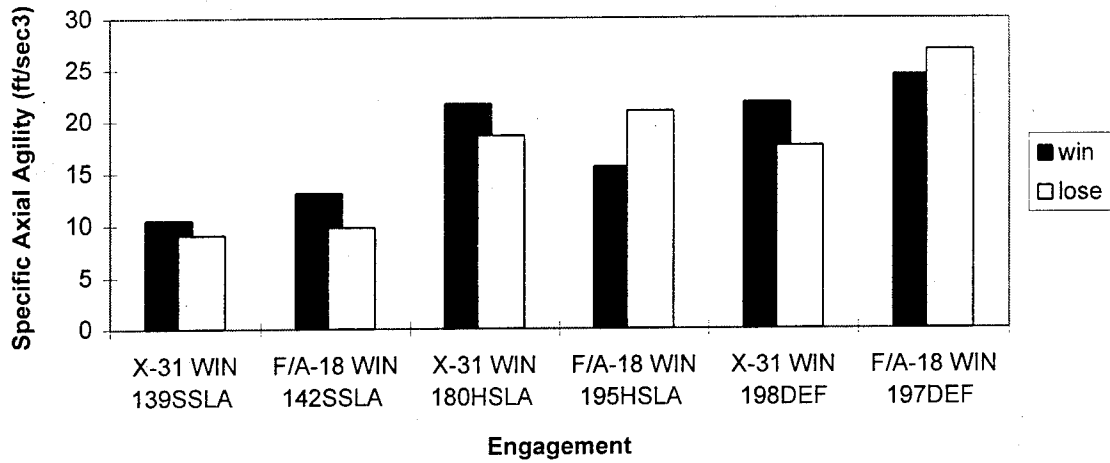


Figure 19. Specific Axial Agility Metric - Partial Engagement Time

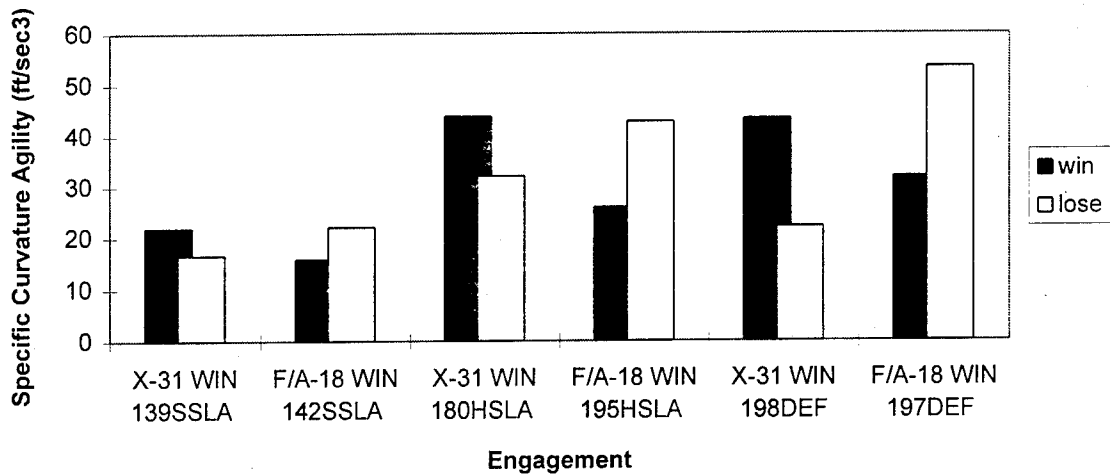


Figure 20. Specific Curvature Agility Metric - Partial Engagement Time

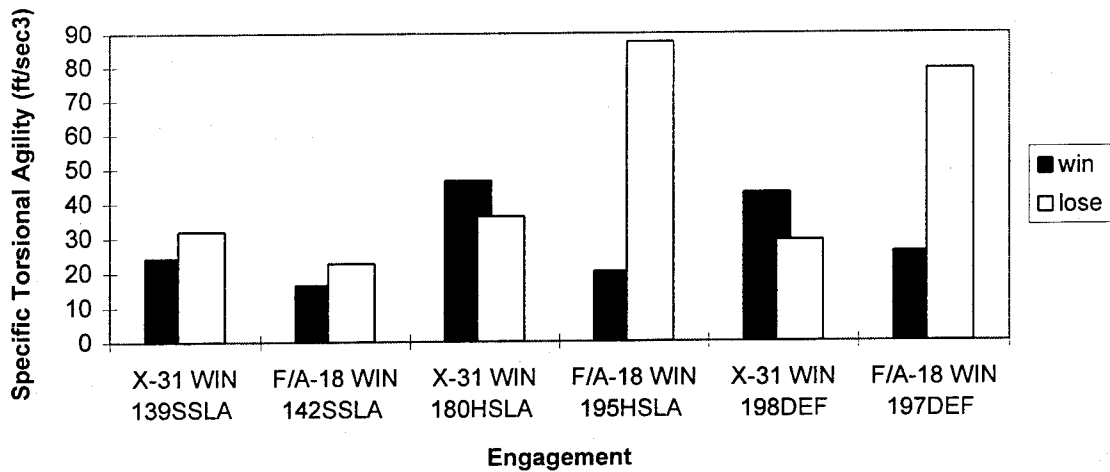


Figure 21. Specific Torsional Agility Metric - Partial Engagement Time

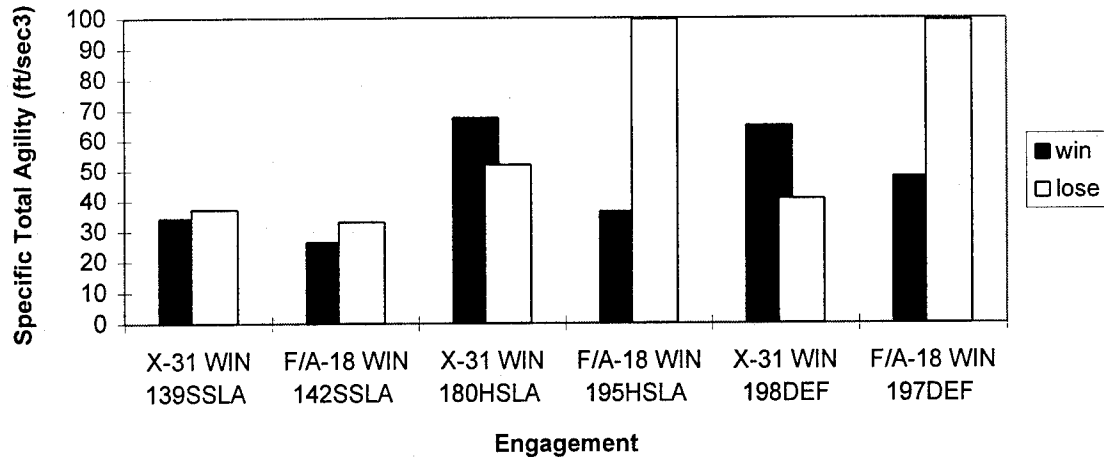


Figure 22. Specific Total Agility Metric - Partial Engagement Time

The Tactical Agility Metrics for the partial engagement times are shown in figures 23 through 26. Only in figure 23 did the Tactical Axial Agility metric show an improvement in the predictive capability of this metric using the partial engagement time. In the remaining figures using the reduced time either provided less cases where the Agility metrics matched or it failed to match those engagements that had the greater value for the different agility metrics. Therefore, reducing the time segment did not improve the predictive capabilities of the Specific and Tactical Agility metrics.

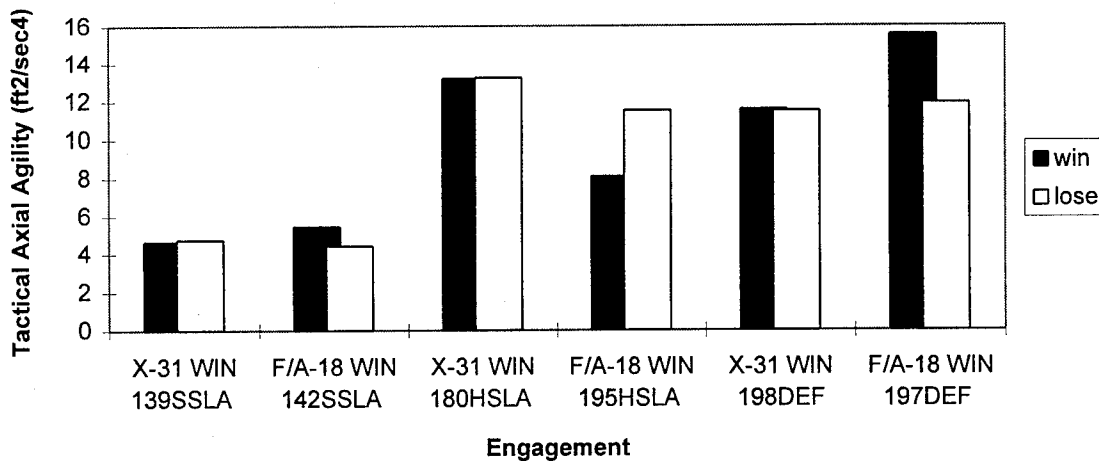


Figure 23. Tactical Axial Agility Metric - Partial Engagement Time

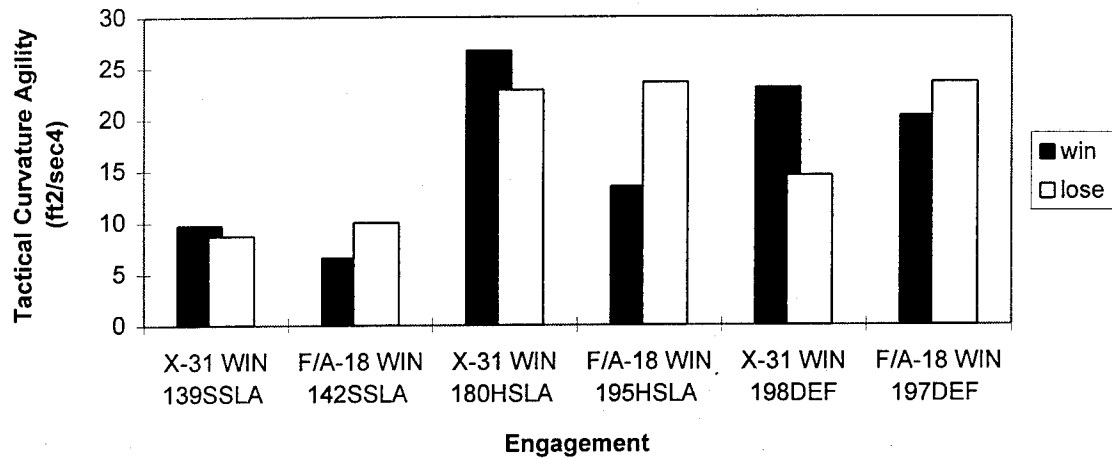


Figure 24. Tactical Curvature Agility Metric - Partial Engagement Time

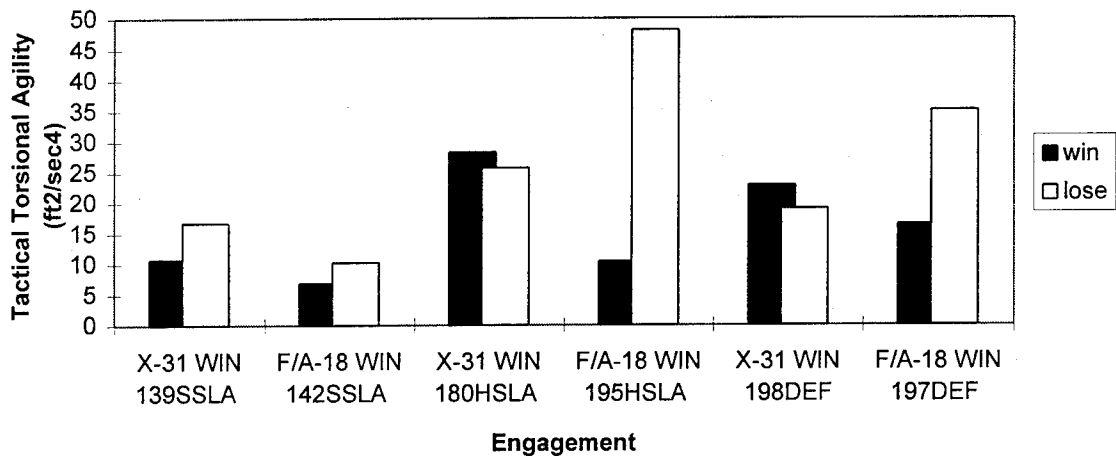


Figure 25. Tactical Torsional Agility Metric - Partial Engagement Time

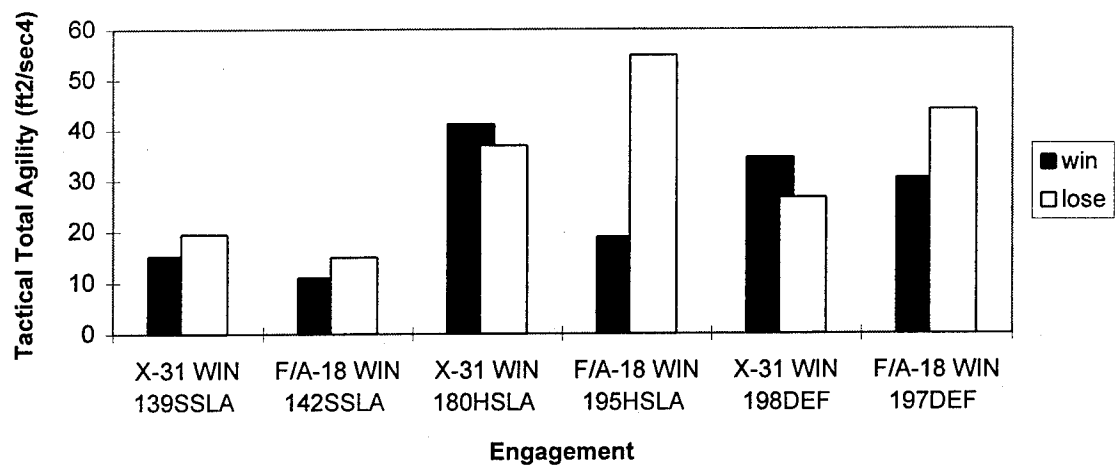


Figure 26. Tactical Total Agility Metric - Partial Engagement Time

These preliminary results do not demonstrate correlation for the Agility Metric as an indicator that the more agile aircraft will get the win. Possible reasons for the mixed results may be: (1) in order to compute the agility metrics the user needs to compute acceleration rate, (2) in actual flight test data, values for acceleration may be extremely noisy, and (3) differentiating with such a noisy input will result in large magnitudes for acceleration rate which in turn inflates the value for the agility metrics. Also, as was previously indicated, there is an average speed term incorporated into the Tactical Agility metrics. Part of the X-31's "agility" is that it can slow down and use thrust vectoring to enhance its pointing capability and thus launch the winning missile. This is in direct contradiction to the supposition in reference 1 that the greater the value for Tactical Agility, the more agile the aircraft, the greater number of wins. Also because of the mixed results one cannot say that less is better.

### 3.2 OBSERVATIONS

The performance plots were reviewed to try to determine if there was some common denominator that would predict when either aircraft would score the win and if it was an "agility" parameter. The following performance parameters were plotted for the 6 CIC engagements studied:  $n_z$ ,  $\dot{n}_z$ , angle-of-attack,  $P_s$ , velocity, bank angle, roll rate, pitch rate, turn rate, off-boresight angle and off-velocity angle. Also considered were various time segments of each of the engagements. There seemed to be two major trends that were observed: (1) once the winning aircraft was able to achieve an off-boresight angle that was within the weapons release angle (approximately halfway through the engagement), then that aircraft was most likely to score the win, and (2) the aircraft that achieved and maintained either the higher  $n_z$  or  $\dot{n}_z$  within the first 5 seconds of the engagement was then in the better position to achieve the weapons resolution angle to obtain a win through a missile kill.

Figures 27 through 32 show the off-boresight angle plots for the 6 CIC engagements studied. It can be seen that in all the engagements except for 197DEF (figure 32), the winning aircraft was near or very near to achieving a weapons firing solution half way through the engagement. In engagement 197DEF the F/A-18 had the advantage early (first 25 seconds) but the X-31 was able to maneuver out of the F-18's firing solution. But the F/A-18 was able to reacquire the weapon's firing solution to obtain the win.

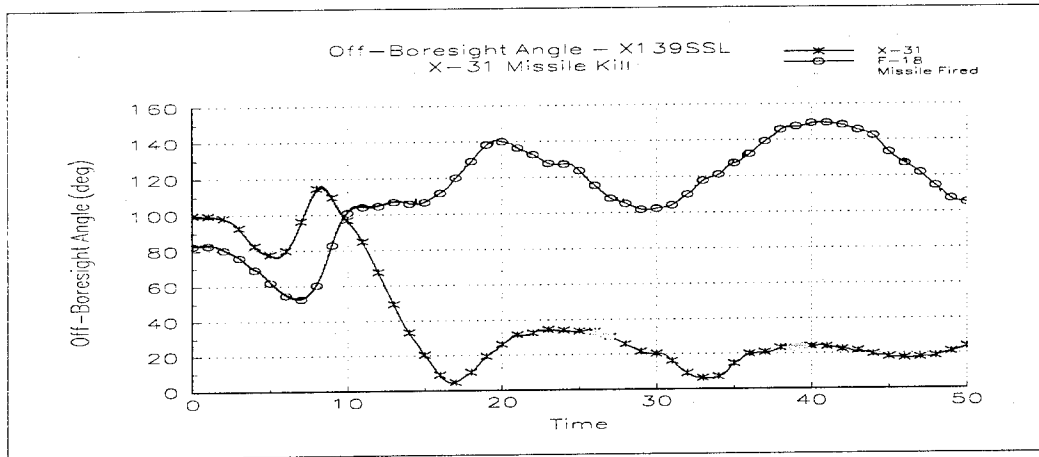


Figure 27. Off-Boresight Angle Plot of Engagement 139SSLA

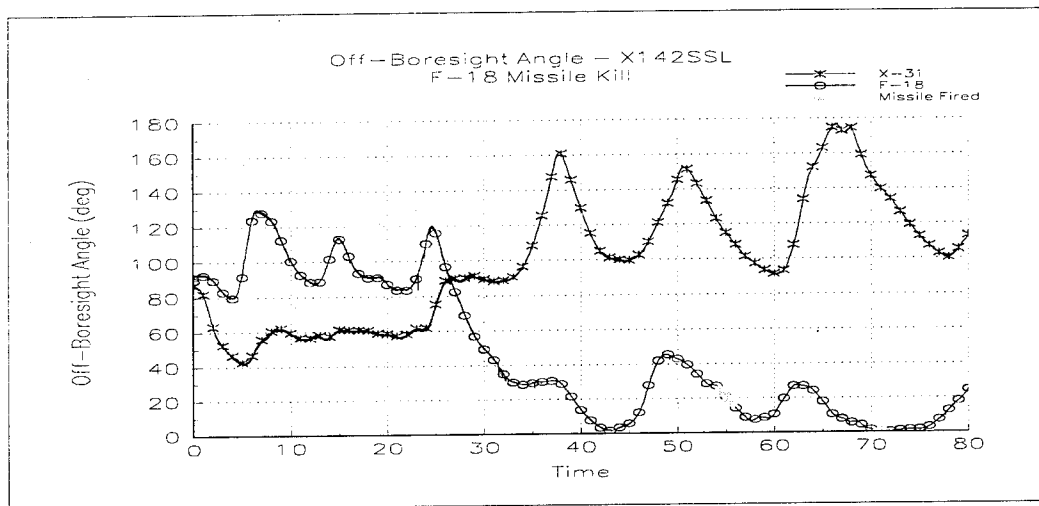


Figure 28. Off-Boresight Angle Plot of Engagement 142SSLA

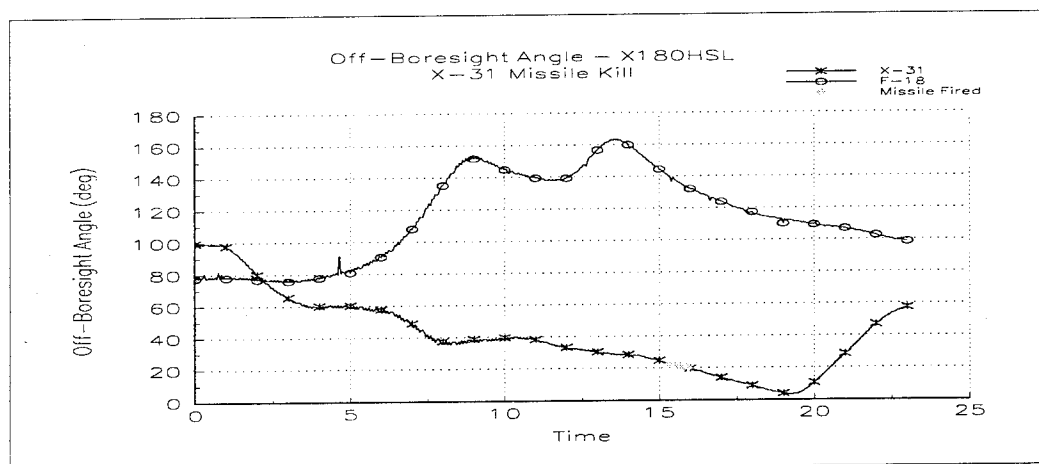


Figure 29. Off-Boresight Angle Plot of Engagement 180HSLA

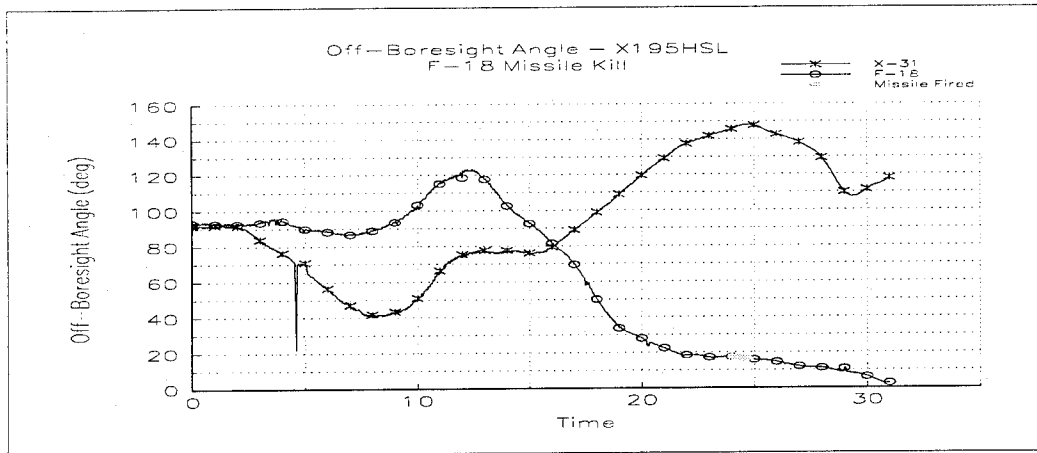


Figure 30. Off-Boresight Angle Plot of Engagement 195HSLA

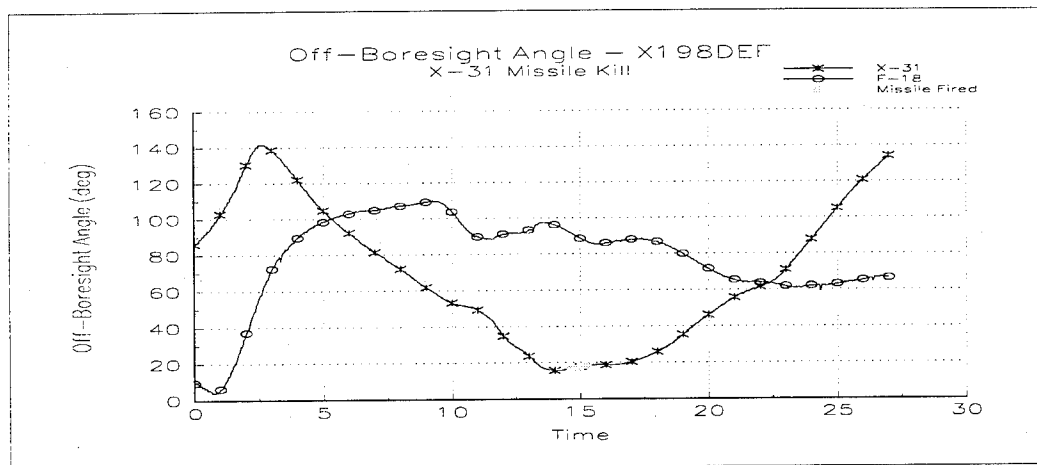


Figure 31. Off-Boresight Angle Plot of Engagement 198DEF

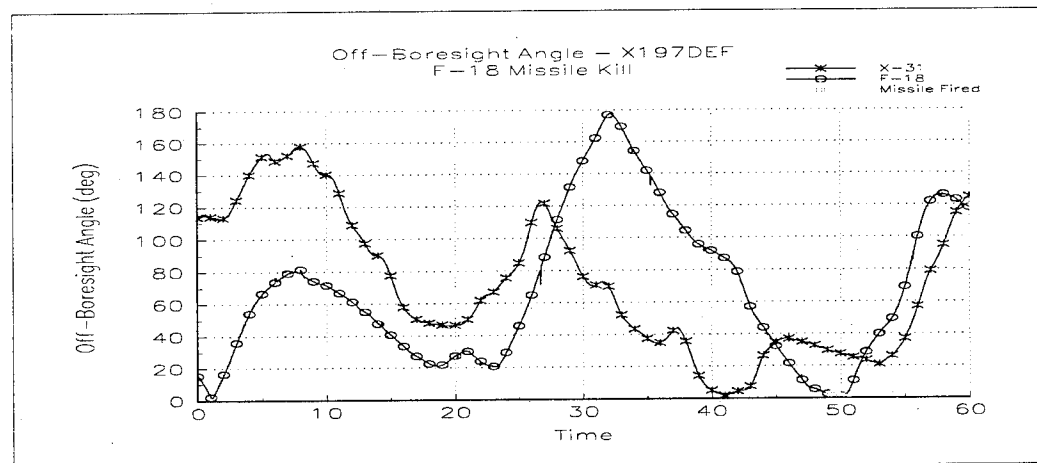


Figure 32. Off-Boresight Angle Plot of Engagement 197DEF



Figures 33 through 38 are the  $n_z$  and  $\dot{n}_z$  plots for the first 5 seconds of the 6 CIC engagements studied. It can be seen that for all the engagements the winning aircraft used  $n_z$  to gain the position advantage. Also, as can be seen in figures 36 and 37 (engagements 195HSLA and 198DEF), where the  $n_z$  values achieved by either aircraft were nearly the same, the aircraft that had the higher  $\dot{n}_z$  value when the aircraft began pulling g's corresponded to the winning aircraft in the final outcome of the engagement. This indicates that the higher initial  $\dot{n}_z$  values allowed the winning aircraft to be in a position to score a missile kill. An additional consideration is whether or not the aircraft is able to maintain the higher  $n_z$  values. In figure 38, engagement 197DEF, the X-31 achieves the higher  $n_z$  during the first 4 seconds of the engagement, but the F/A-18 was able to generate a higher  $n_z$  value by 5 seconds into the engagement and maintain the  $n_z$  advantage throughout the remainder of the engagement as is shown in figure 39.

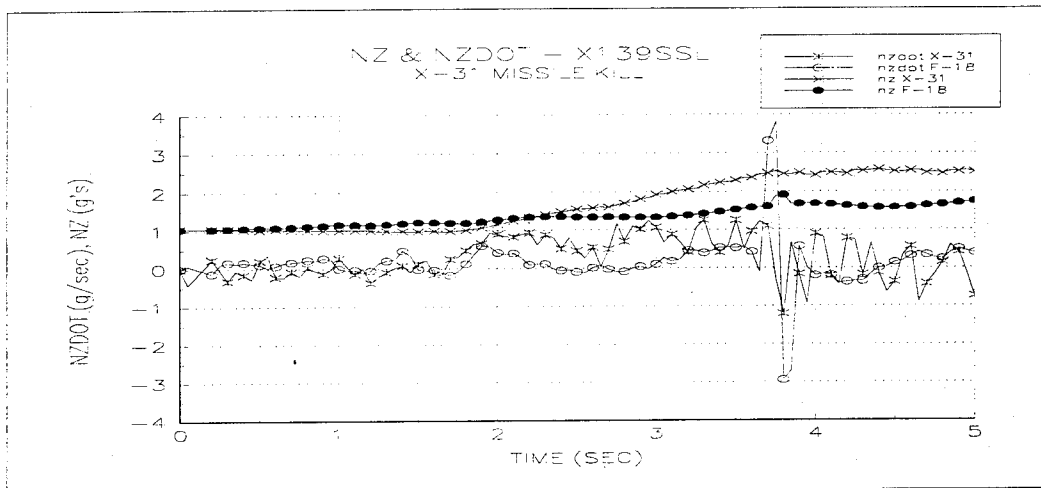


Figure 33.  $N_z$  &  $\dot{N}_z$  Plot of Engagement 139SSLA

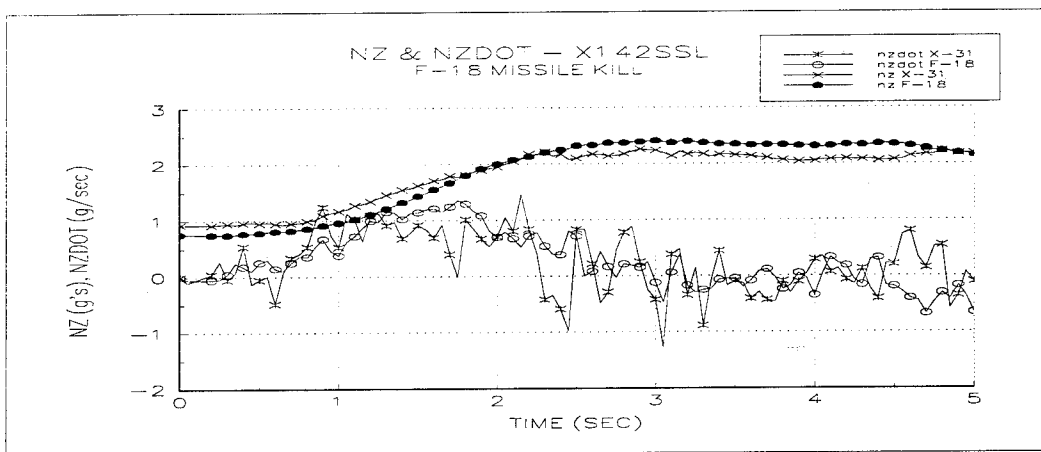
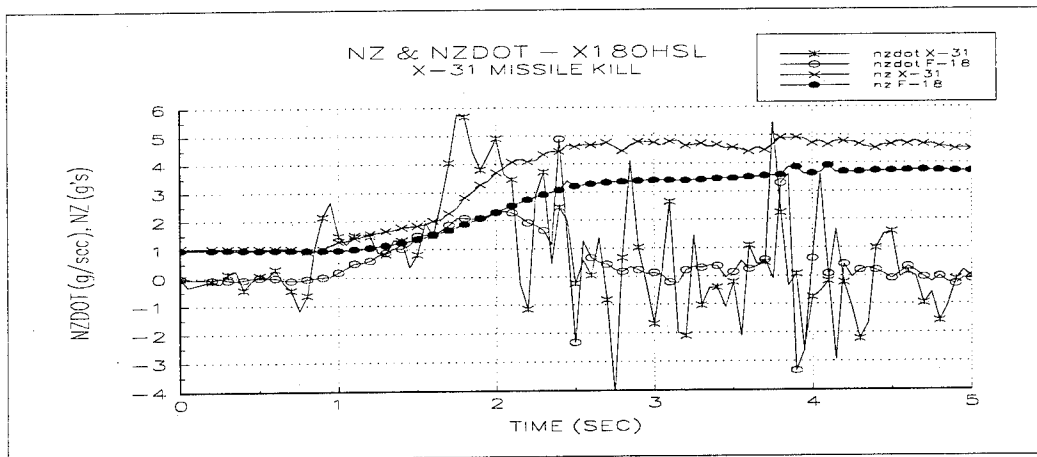
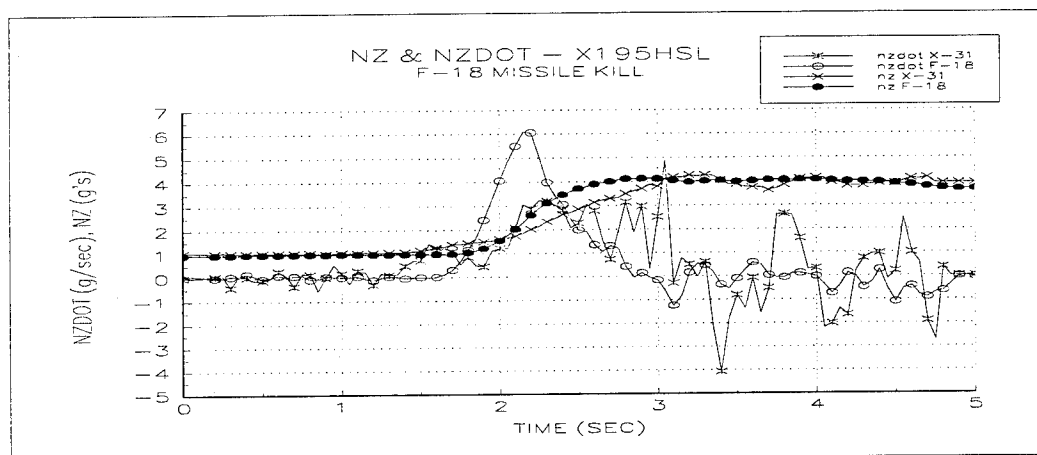
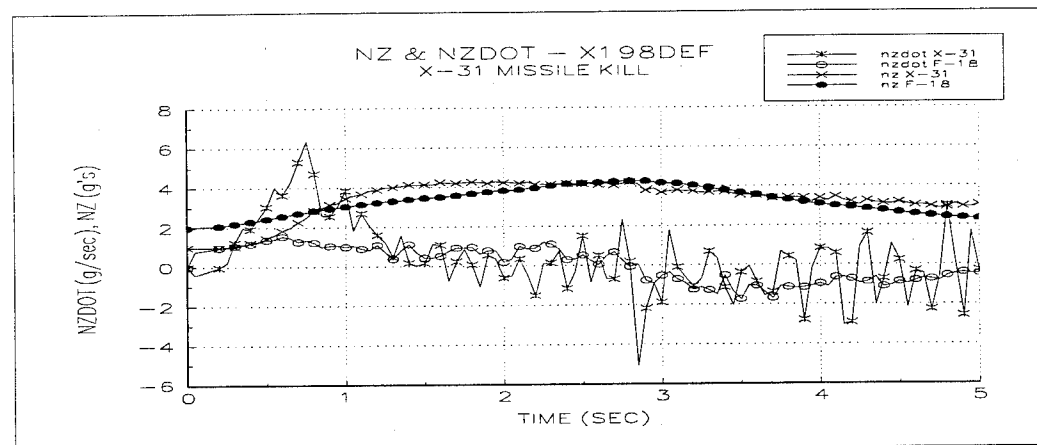
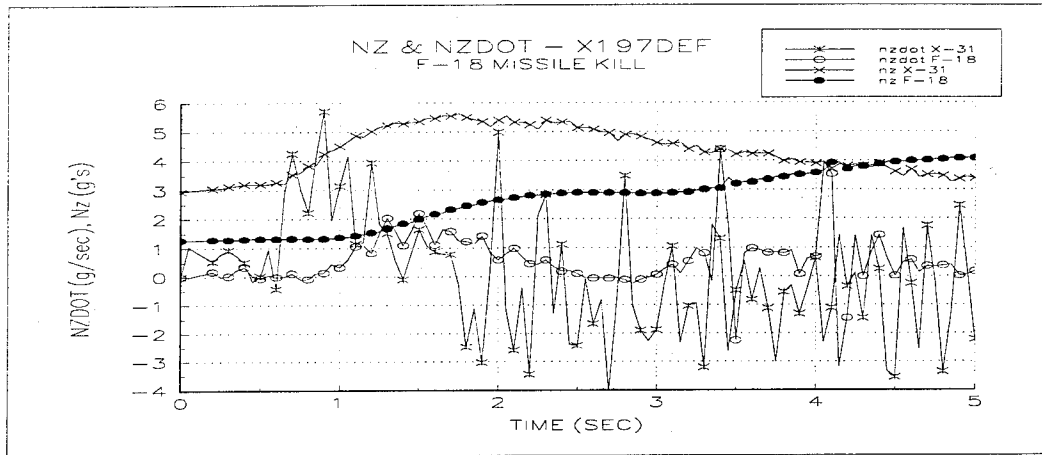
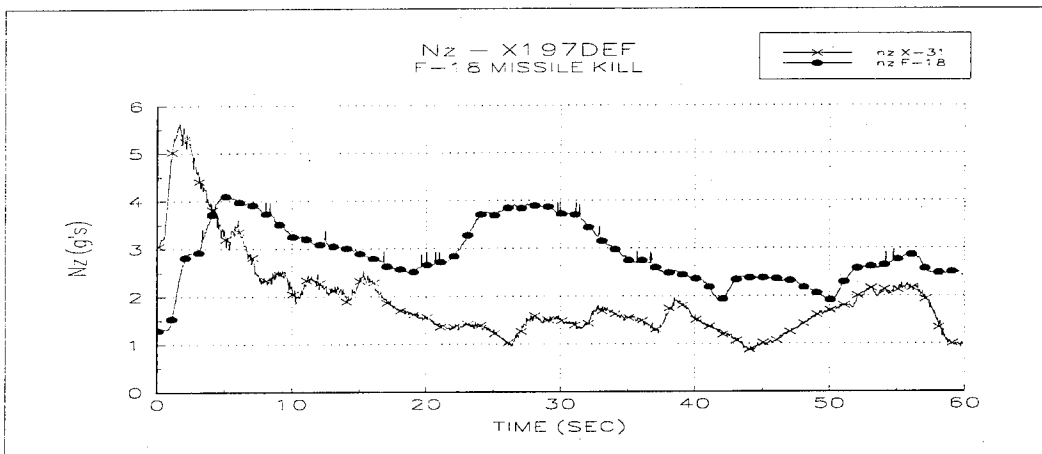


Figure 34.  $N_z$  &  $\dot{N}_z$  Plot of Engagement 142SSLA

Figure 35.  $N_z$  &  $\dot{N}_z$  Plot of Engagement 180HSLAFigure 36.  $N_z$  &  $\dot{N}_z$  Plot of Engagement 195HSLAFigure 37.  $N_z$  &  $\dot{N}_z$  Plot of Engagement 198DEF

Figure 38.  $N_z$  &  $\dot{N}_z$  Plot of Engagement 197DEFFigure 39.  $N_z$  Plot of Engagement 197DEF for Full Engagement

These results are in agreement with reference 5 which tried to determine agility metrics for air combat effectiveness. In reference 5,  $n_z$  and  $\dot{n}_z$  were cited as one of the top factors that determined tactical advantage when position was used to gain the tactical advantage, as was indicated in the results for the engagements evaluated in this study. Of course other factors contribute toward obtaining a position tactical advantage such as:  $q_{\max}$ ,  $p_{\max}$ ,  $r_{\max}$ ,  $P_{s_{\max/\min}}$ , nose rate $_{\max}$ , torsion rate $_{\max}$ ,  $n_{z_{\min}}$ , and  $\dot{n}_{z_{\min}}$ . For other engagements the above cited terms may be more critical in determining the outcome of an engagement.

Reference 6 used a similar approach to defining agility and it was found in references 7 and 8 that  $\dot{n}_z$  can be used to define the curvature component of agility,  $\dot{n}_x$  can be used to describe the axial component of agility and either  $\dot{n}_y$  or  $\dot{p}_{\text{stab}}$  can be used to describe the torsional component of agility. This indicates that highly "agile" aircraft have the ability to effect high pitch rates, large roll accelerations and fast engine response transients.

#### 4.0 CONCLUSIONS

For the six CIC engagements investigated in this study the agility metrics presented in reference 1 were not able to consistently predict which aircraft would score a win. A review of the performance plots showed that the aircraft that had either the higher initial  $n_z$ , or higher  $\dot{n}_z$  within the first 5 seconds of the engagement was most likely to score a win. This is in agreement with other studies which have tried to quantify agility win combat effectiveness. Reference 1 attempted to expand on the acceleration rate vectors by defining the Tactical and Specific Agility metrics in terms of acceleration rate components ( $\dot{a}_{b,n,t}$ ), inflection points ( $\eta$ ), distance traveled ( $S$ ), and time of engagement ( $\Delta t$ ). However, results of this analysis indicate that by just evaluating the axial, torsional and curvature components of the acceleration rate vector (as was done in references 6, 7, and 8) these "agility" terms may be more useful in defining agility's role in combat effectiveness.

#### 5.0 RECOMMENDATIONS

This study only looked at a small sample of the flights between the X-31 and the F/A-18. It would be interesting to review additional engagements to see if the acceleration rate vector components as presented in either references 7 or 8, would provide additional insight to agility and its application to combat effectiveness. In addition to the X-31 and F/A-18 engagements, results from other engagements between different aircraft such as those in references 5 or 9 would be beneficial in determining the effect of variation of aircraft design on agility and its application to combat effectiveness.

## REFERENCES

1. Mazza, C. J., "Aircraft Fighter Agility: Theory, Advanced Metrics, and Simulation", Veda Technical Report 33158-94U/P3663-003, July 14, 1994.
2. Brand, L., "Vector Analysis", John Wiley & Sons, 1957.
3. Canter, David E. and Groves, Allen W. , "X-31 Tactical Utility - Initial Results", 1993.
4. Lee, Bryson, "CIC Engagement Data Files", Memorandum dated 27 April 1994.
5. Shaw, Robert L., "Agility Metrics ACMI Combat Engagement Effectiveness Analysis", Report No. WL-TR-93-3017, January 1993.
6. Herbst, W., et al, "Evaluation of Agility for Future Aircraft", MBB/LKE/X-31A/R-29, 30 November 1987.
7. Glenister, R.L., " A Critical Evaluation of MBB/Rockwell Agility Metrics", Rockwell TFD-88-1429, 30 September 1988.
8. Booz, J.E., "Reformulation of the Acceleration Rate Vector From Frenet's Formulas using Body Axis Velocities and Accelerations", AIAA-94-3471, August 1994.
9. Wilson, D.J., "Aircraft Maneuvers for the Evaluation of Flying Qualities and Agility - Maneuver Descriptions and Selection Guide", Report No. WL-TR-93-3083.

APPENDIX A

FRENET'S FORMULAS AS APPLIED TO THE  
ACCELERATION RATE VECTORS

The acceleration rate vector is developed from the following set of equations for Frenet's formulas. Vectors are denoted by bold letters.

$$\mathbf{R} = \text{Flight Path} = R(s) \quad (1)$$

$$\dot{\mathbf{R}} = \text{Velocity} = \dot{s} \mathbf{t} = U \mathbf{i}_b + V \mathbf{j}_b + W \mathbf{k}_b \quad (2)$$

$$\ddot{\mathbf{R}} = \text{Acceleration} = \ddot{s} \mathbf{t} + \dot{s}^2 \kappa \mathbf{n} = a_x \mathbf{i}_b + a_y \mathbf{j}_b + a_z \mathbf{k}_b \quad (3)$$

$$\begin{aligned} \ddot{\mathbf{R}} = \text{Accel. Rate} &= (\ddot{s} - \dot{s}^3 \kappa^2) \mathbf{t} + (3\dot{s}\ddot{s} + \dot{s}^2 \dot{\kappa}) \mathbf{n} + \dot{s}^3 \kappa \tau \mathbf{b} \\ &= \dot{a}_x \mathbf{i}_b + \dot{a}_y \mathbf{j}_b + \dot{a}_z \mathbf{k}_b \end{aligned} \quad (4)$$

where:

- $\dot{s}$  = first derivative of the flight path wrt time (ft/sec)
- $\ddot{s}$  = second derivative of the flight path wrt time (ft/sec<sup>2</sup>)
- $\dddot{s}$  = third derivative of the flight path wrt time (ft/sec<sup>3</sup>)
- $\kappa$  = curvature = 1/turn radius (1/ft)
- $\dot{\kappa}$  = curvature rate = 1/turn radius rate (1/ft\*sec)
- $\tau$  = torsion (1/ft)
- $\mathbf{t}$  = tangential unit vector
- $\mathbf{n}$  = normal unit vector
- $\mathbf{b}$  = binormal unit vector
- $U$  = x-component of body axis velocity (ft/sec)
- $V$  = y-component of body axis velocity (ft/sec)
- $W$  = z-component of body axis velocity (ft/sec)
- $a_x$  = x-component of body axis acceleration (ft/sec<sup>2</sup>)
- $a_y$  = y-component of body axis acceleration (ft/sec<sup>2</sup>)
- $a_z$  = z-component of body axis acceleration (ft/sec<sup>2</sup>)
- $\dot{a}_x$  = x-component of body axis acceleration rate (ft/sec<sup>3</sup>)
- $\dot{a}_y$  = y-component of body axis acceleration rate (ft/sec<sup>3</sup>)
- $\dot{a}_z$  = z-component of body axis acceleration rate (ft/sec<sup>3</sup>)
- $\mathbf{i}_b$  = body axis unit vector in x direction (+ forward)
- $\mathbf{j}_b$  = body axis unit vector in y direction (+ right wing)
- $\mathbf{k}_b$  = body axis unit vector in z direction (+ down)

The components of the acceleration rate vector can then be broken down into the following components:

$$\dot{a}_a = \text{axial component} = (\ddot{s} - \dot{s}^3 \kappa^2) \quad (5)$$

$$\dot{a}_c = \text{curvature component} = (3\dot{s}\ddot{s} + \dot{s}^2 \dot{\kappa}) \quad (6)$$

$$\dot{a}_t = \text{torsional component} = \dot{s}^3 \kappa \tau \quad (7)$$



APPENDIX B  
TABULATED DATA

Table B-1 Specific Agility Values for Full Engagement Time					
Engagement	kill/lose	Axial	Curvature	Torsional	Total
139SSL - X-31	kill	12.6212	25.1539	33.5291	43.7747
139SSL - F/A18	lose	16.9989	16.4008	26.6745	35.6298
142SSL- X-31	lose	14.338	34.8445	42.7273	56.9678
142SSL - F/A-18	kill	16.7067	16.9398	14.7801	28.0092
180HSL - X-31	kill	21.6562	43.7965	46.4148	67.3906
180HSL - F/A-18	lose	18.6156	32.0741	36.0953	51.7512
195HSL - X-31	lose	20.7942	42.6706	86.9417	99.0558
195HSL - F/A-18	kill	15.3819	25.5897	19.9248	35.8945
197DEF - X-31	lose	22.4288	45.1244	61.7866	79.7299
197DEF - F/A-18	kill	22.4252	28.5023	23.9737	43.4742
198DEF - X-31	kill	17.0145	34.0004	33.1866	50.4665
198DEF - F/A-18	lose	15.5067	20.6383	22.8318	34.4625

Table B-2 Tactical Agility Values for Full Engagement Time					
Engagement	kill/lose	Axial/1000	Curvature/1000	Torsional/1000	Total/1000
139SSL - X-31	kill	5.6941	11.3484	15.1269	19.7493
139SSL - F/A18	lose	8.4131	8.1171	13.2018	17.634
142SSL- X-31	lose	6.5958	16.0293	19.6556	26.2066
142SSL - F/A-18	kill	7.0377	7.1359	6.2261	11.7989
180HSL -X-31	kill	13.205	26.7052	28.3017	41.0919
180HSL - F/A-18	lose	13.2542	22.8365	25.6996	36.8465
195HSL - X-31	lose	11.3829	23.3583	47.5926	54.224
195HSL - F/A-18	kill	7.9326	13.1969	10.2754	18.5112
197DEF - X-31	lose	8.5297	17.1609	23.4976	30.3215
197DEF - F/A-18	kill	12.1833	15.4849	13.0246	23.6189
198DEF - X-31	kill	7.2807	14.5491	14.2009	21.5951
198DEF - F/A-18	lose	8.3726	11.1434	12.3278	18.6076

Table B-3 Specific Agility Values for Partial Engagement Time					
Engagement	kill/lose	Axial	Curvature	Torsional	Total
139SSL - X-31	kill	10.4852	21.8325	24.1862	34.2284
139SSL - F/A18	lose	9.0815	16.581	31.8895	37.0721
142SSL - X-31	lose	9.8055	22.0747	22.5954	33.0754
142SSL - F/A-18	kill	13.0611	15.7564	16.4534	26.2597
180HSL - X-31	kill	21.6562	43.7965	46.4148	67.3906
180HSL - F/A-18	lose	18.6156	32.0741	36.0953	51.7512
195HSL - X-31	lose	20.9523	42.7662	87.3774	99.5126
195HSL - F/A-18	kill	15.6001	25.8945	20.146	36.3282
197DEF - X-31	lose	26.9091	53.4365	79.3158	99.3507
197DEF - F/A-18	kill	24.4394	31.938	25.8468	47.8055
198DEF - X-31	kill	17.0145	34.0004	33.1866	50.4665
198DEF - F/A-18	lose	15.5067	20.6383	22.8318	34.4625

Table B-4 Tactical Agility Values for Partial Engagement Time					
Engagement	kill/lose	Axial/1000	Curvature/1000	Torsional/1000	Total/1000
139SSL - X-31	kill	4.6577	9.6983	10.7439	15.2048
139SSL - F/A18	lose	4.7622	8.6948	16.7223	19.44
142SSL - X-31	lose	4.4373	9.9896	10.2252	14.9678
142SSL - F/A-18	kill	5.4409	6.5637	6.854	10.9391
180HSL -X-31	kill	13.205	26.7052	28.3017	41.0919
180HSL - F/A-18	lose	13.2542	22.8365	25.6996	36.8465
195HSL - X-31	lose	11.5387	23.552	48.1199	54.803
195HSL - F/A-18	kill	8.0807	13.4131	10.4354	18.8176
197DEF - X-31	lose	11.9149	23.6607	35.1196	43.9907
197DEF - F/A-18	kill	15.5389	20.3067	16.4338	30.3955
198DEF - X-31	kill	7.2807	14.5491	14.2009	21.5951
198DEF - F/A-18	lose	15.5389	20.3067	16.4338	30.3955

Veda, Inc.  
730 Louis Drive  
Warminster, PA 18974  
(Attn: J. Kern)

NASA Langley Research Center Hampton, VA 30350 (Attn: J. Batterson M.S. 498) (Attn: M. Ogburn M.S. 355) (Attn: L. Nguyen M.S. 355)	3
Rockwell, International P. O. Box 90928 El Segundo, CA 90009 (Attn: B. F. Tamrat)	1
General Dynamics Fort Worth P. O. Box 748 Fort Worth, TX 76101 (Attn: M. Yokell Mail Zone 2872)	1
Lockheed Aero Systems Company 86 South Cobb Drive Marietta, GA 30063-0685 (Attn: Dr. H. Youssef Dept 73-42)	1
Eidetics International 3415 Lomita Blvd Torrance, CA 90505 (Attn: A. Skow)	1
Fighter Command International 1831 Stonewood Drive Beavercreek, OH 45432 (Attn: R. Shaw)	1
McDonnell Douglas P. O. Box 516 St. Louis, MO 63166-0516 (Attn: K. Citurs) (Attn: D. Wilson)	2
Northrop Grumman Corp 1 Northrop Ave Hawthorne, CA 90250-3277 (Attn: J. Kalviste)	1
Systems Technology, Inc. 13766 South Hawthorne Boulevard Hawthorne, CA 90250 (Attn: T. Myers)	1

**DISTRIBUTION LIST**  
**Report No. NAWCADWAR-95012-4.3**

	No. of Copies
AFWAL Wright-Patterson AFB, OH 45433 (Attn: Mr. T. Cord, Code FIGC)	1
Center for Naval Analysis 4401 Fort Avenue PO BOX 16268 Alexandria, VA 22304-0268	1
Defense Technical Information Center Cameron Station BG5 Alexandria, VA 22304-6145 (ATTN: DTIC-FDAB)	2
Naval Air Systems Command Washington, DC 20361-0001 (2 copies for Code 432000W) (2 copies for Library)	4
Naval Air Warfare Center Aircraft Division Warminster, PA 18974-0591 (2 copies for Code 43C000R08) (1 copy for Code 431000R08) (1 copy for Code 430000R08) (1 copy for Code 432000R08) (15 copies for Code 432300R08; J. Booz) (2 copies for Code 725500R68)	22
Office of Naval Research 800 N. Quincy St., Arlington, VA 22217-5000 (2 copies for Code 212; W. King)	2
NASA Dryden Research Center 412 TW/TSFF 195 South Popson Ave. Edwards, CA 93524-6842 (Attn: Capt. Bryson Lee )	1